HEAT EXCHANGE IN TUBES WITH PERMEABLE WALLS IN THE PRESENCE OF INTERNAL HEAT SOURCES

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Solutions are obtained for the energy equation for fluid flow in tubes with uniform injection of suction and heat sources in the stream.

Problems about the heat exchange in tubes with injection or suction through permeable walls in the absence of heat liberation in the flow have been examined in a number of papers, e.g., for boundary conditions of the first kind in [1-4], and for boundary conditions of the second kind in [3, 5]. Interest in these problems is evoked principally by the extensive utilization of injection in the interests of heat shielding, and suction for the intensification of heat emission.

Solutions are obtained in this paper for the energy equation in the presence of internal heat sources distributed uniformly over the tube length in the case of boundary conditions of the first and second kinds. It is assumed that the flow is stabilized hydrodynamically and the coefficient of effective heat conduction γ does not vary along the tube length, the axial heat conduction is negligibly small, and the physical properties of the fluid are constant. Taking account of the assumptions made, the energy equation is written in the form

$$c_p\rho\left(u_x\frac{\partial T}{\partial x}+u_r\frac{\partial T}{\partial r}\right)=\frac{\lambda}{r^\alpha}\frac{\partial}{\partial r}\left(r^\alpha\gamma\left(\frac{r}{r_0}\right)\frac{\partial T}{\partial r}\right)+\bar{q}_v f\left(\frac{r}{r_0}\right), \tag{1}$$

where $\bar{q}_v=2^\alpha\int\limits_0^1 q_v\eta^\alpha d\eta$ is the mean density of the heat sources in a tube section.

The axial and radial velocity components for a hydrodynamically stabilized flow can be expressed in terms of one function $F(\eta;\,\mathrm{Re}_V)$ [6, 7]

$$u_x=\left(U_0-\frac{2^\alpha V_x}{r_0}\right)\frac{F'(\eta)}{(2\eta)^\alpha},\quad u_r=V\frac{F(\eta)}{\eta^\alpha}; \tag{2}$$

where F satisfies the boundary conditions

$$\lim_{\eta\to 0}\frac{F(\eta)}{\eta^\alpha}=0,\ \lim_{\eta\to 0}\left(\frac{F'(\eta)}{\eta^\alpha}\right)=0,\ F(1)=1,\ F'(1)=0.$$

Taking account of (2) in the dimensionless variables

$$\theta=\frac{T\lambda}{\bar{q}_v r_0^2},\quad \eta=\frac{r}{r_0},\quad Z=-\frac{2}{\mathrm{Pe}_v}\ln\left(1-\frac{2^\alpha xV}{r_0U_0}\right),\ \mathrm{Pe}_v=\frac{2r_0V\rho c_p}{\lambda}$$

we obtain from (1)

$$F'(\eta)\frac{\partial\theta}{\partial Z}+\frac{\mathrm{Pe}_v}{2}F(\eta)\frac{\partial\theta}{\partial\eta}=\frac{\partial}{\partial\eta}\left(\eta^\alpha\gamma(\eta)\frac{\partial\theta}{\partial\eta}\right)+\eta^\alpha f(\eta). \tag{3}$$

Let us examine the solution of (3) under boundary conditions of the first kind by considering the temperature of the fluid at the entrance to the tube and at the walls to be constant and equal to T_0 and T_W, respectively:

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$$\theta(0,\ \eta)=\theta_0=\frac{T_0\lambda}{\bar{q}_v r_0^2},\ \left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}=0,\ \theta(Z,\ 1)=\theta_w=\frac{T_w\lambda}{\bar{q}_v r_0^2}. \tag{4}$$

Let us represent the solution of (3) with the boundary conditions (4) in the form

$$\theta(z,\ \eta)=\theta_*(\eta)+\theta_1(Z,\ \eta), \tag{5}$$

where $\theta_*(\eta)$ is the solution of the problem in the thermal stabilization domain for large values of Z

$$\frac{\partial}{\partial\eta}\left(\eta^\alpha\gamma(\eta)\ \frac{\partial\theta_*}{\partial\eta}\right)-\frac{\mathrm{Pe}_v}{2}F(\eta)\frac{\partial\theta_*}{\partial\eta}+\eta^\alpha f(\eta)=0, \tag{6}$$

$$\left(\frac{\partial\theta_*}{\partial\eta}\right)_{\eta=0}=0,\ \theta_*(1)=\theta_w.$$

To determine θ_1 we obtain the equation

$$F'(\eta)\frac{\partial\theta_1}{\partial z}-\frac{\mathrm{Pe}_v}{2}F(\eta)\frac{\partial\theta_1}{\partial\eta}=\frac{\partial}{\partial\eta}\left(\eta^\alpha\gamma(\eta)\ \frac{\partial\theta_1}{\partial\eta}\right) \tag{7}$$

with the boundary conditions

$$\theta_1(0,\ \eta)=\theta_0-\theta_*(\eta),\ \left(\frac{\partial\theta_1}{\partial\eta}\right)_{\eta=0}=0,\ \theta_1(Z,\ 1)=0.$$

The solution of (7), analogous to the solutions of a problem on heat exchange in tubes with suction and injection in the absence of internal heat sources considered in [1-4], has the form

$$\theta_1=\sum_{n=0}^{\infty}A_n\varphi_n(\eta)\exp(-\varepsilon_n^2 Z), \tag{8}$$

where ε_n and φ_n are the eigenvalues and eigenfunctions of the following problem:

$$\frac{d}{d\eta}\left(\eta^\alpha\gamma(\eta)\ \frac{d\varphi_n}{d\eta}\right)-\frac{\mathrm{Pe}_v}{2}F(\eta)\frac{d\varphi_n}{d\eta}+\varepsilon_n^2F'(\eta)\varphi_n=0,\ \varphi_n'(0)=\varphi_n(1)=0, \tag{9}$$

and the constants A_n are determined by the expression

$$A_n=\frac{\int_0^1(\theta_0-\theta_*)F'\Phi(\eta)\varphi_n d\eta}{\int_0^1 F'\Phi(\eta)\varphi_n^2 d\eta}=\frac{2}{\varepsilon_n\left(\frac{\partial\varphi}{\partial\varepsilon}\right)_{\substack{\eta=1\\ \varepsilon=\varepsilon_n}}}\left[\theta_0+\frac{\int_0^1\eta^\alpha f\Phi(\eta)\varphi_n d\eta}{\Phi(1)\left(\frac{d\varphi_n}{d\eta}\right)_{\eta=1}}\right],$$

where

$$\Phi(\eta)=\exp\left(-\frac{\mathrm{Pe}_v}{2}\int_0^\eta\frac{Fd\lambda}{\lambda^\alpha\gamma}\right).$$

The solution of (6) will be

$$\theta_*=\theta_w+\int_\eta^1\frac{d\lambda}{\Phi(\lambda)\lambda^\alpha\gamma}\int_0^\lambda\xi^\alpha f\Phi(\xi)\,d\xi. \tag{10}$$

For $\mathrm{Pe}_V=0$ we obtain the temperature distribution in a tube with impermeable walls [8] from (10).

Let us determine the Nusselt number $\mathrm{Nu}=-\frac{2}{\theta_m}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=1}$, where $\theta_m=\int_0^1\theta F'd\eta$ is the average mass stream temperature. Using (10), we obtain the following formula for Nu in the stabilized heat-transfer domain:

$$\mathrm{Nu}_*=\mathrm{Pe}_v+2\Phi(1)\left[\int_0^1 F\Phi(\eta)\int_0^\eta(2\lambda)^\alpha fd\lambda\frac{d\eta}{\gamma\eta^\alpha}\right]^{-1}. \tag{11}$$

As an illustration, let us present the results of a computation using (10) and (11) for laminar flow in the case of a uniform distribution of internal heat sources over the tube section for fluid flow with high Prandtl numbers (small Re_V values, respectively), when the flow hydrodynamics is described by a Poiseuille velocity profile. Then

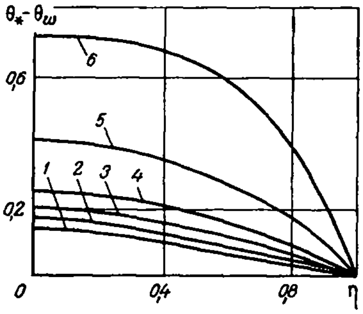

Fig. 1

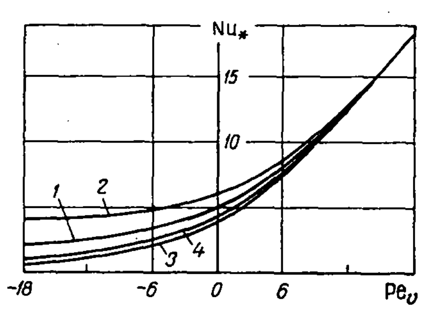

Fig. 2

Fig. 1. Temperature distribution over a circular pipe section for a constant wall temperature: 1) $Pe_V = -16$; 2) -8; 3) -4; 4) 0; 5) 4; 6) 8.

Fig. 2. Dependence $Nu_*(Pe_V)$ in the presence and absence of internal heat sources.

$$\gamma = 1, \; f = 1, \; F(\eta) = \frac{(2\eta)^\alpha}{2}\,[(3-\alpha)\,\eta - \eta^3]. \tag{12}$$

Temperature profiles in a circular tube for different values of the parameter Pe_V characterizing the intensity of the injection or suction are presented in Fig. 1. It is seen from Fig. 1 that injection ($Pe_V < 0$) diminishes for a given wall temperature, while suction ($Pe > 0$) increases the temperature in the flow. The dependence of the Nusselt number on the parameter Pe_V is represented in Fig. 2 for a plane (curve 1) and circular (curve 2) tube. As the suction velocity increases the Nusselt number grows, and the injection diminishes, which agrees with the nature of the behavior in the absence of internal heat sources [1-4]. Let us note that Nu_* tends to the limit values

$$\lim_{Pe_\upsilon \to \infty} Nu_* = Pe_\upsilon, \; \lim_{Pe_\upsilon \to -\infty} Nu_* = 2^{1+\alpha} f_1(1)$$

as the suction and injection intensity increase.

The limit dependence for suction is the same as for $q_V = 0$, while the analogy is spoiled for injection since $\lim_{Pe_\upsilon \to -\infty} Nu_* = 0$ according to [2-4].

In the case of boundary conditions of the second kind, we assume that the total heat flux through the wall q_W is known, which consists of conductive and convective flows in the presence of blowing or suction. Then the first two boundary conditions in (4) remain valid, and we use the one-dimensional energy conservation equation following from (3):

$$\frac{d\theta_m}{dZ} = \frac{Pe_\upsilon}{2}(\theta_m - \theta_w) + \left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=1} + \frac{1}{2^\alpha} = \frac{Q+1}{2^\alpha} \tag{13}$$

as the third condition, where

$$Q = 2^\alpha\left[\frac{Pe_\upsilon}{2}(\theta_m - \theta_w) + \left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=1}\right] = \frac{2^\alpha q_w}{q_\upsilon r_0}.$$

We represent the solution of (3) in a form analogous to (5), where we now obtain for the section of developed heat transfer with (13) taken into account

$$\theta_* = \theta_m(Z) + \psi(\eta) = \theta_0 + \frac{(Q+1)Z}{2^\alpha} + \psi(\eta). \tag{14}$$

The function ψ is found from the solution of the equation

$$\frac{d}{d\eta}\left(\eta^\alpha \gamma(\eta)\frac{d\psi}{d\eta}\right) - \frac{Pe_\upsilon}{2}F(\eta)\frac{d\psi}{d\eta} = \frac{(Q+1)F'(\eta)}{2^\alpha} - \eta^\alpha f(\eta) = f_1(\eta) \tag{15}$$

with the boundary conditions

$$\left(\frac{d\psi}{d\eta}\right)_{\eta=0} = 0, \; \left(\frac{d\psi}{d\eta}\right)_{\eta=1} - \frac{Pe_\upsilon}{2}\psi(1) = \frac{Q}{2^\alpha}.$$

The equation to determine θ_1 agrees with (7) but the boundary conditions will be

$$\theta_1(0, \eta) = -\psi(\eta), \ \left(\frac{\partial \theta_1}{\partial \eta}\right)_{\eta=0} = 0, \ \left(\frac{\partial \theta_1}{\theta \eta}\right)_{\eta=1} = \frac{\mathrm{Pe}_v}{2}\theta_1(Z, 1). \tag{16}$$

Taking account of (16), expression (8), in which ε_n and φ_n are determined from (9) for the boundary conditions $\varphi_n'(0) = 0$, $\varphi_n'(1) = \mathrm{Pe}_v \varphi_n(1)/2$, and the constants A_n are expressed by the formula

$$A_n = -\frac{\int_0^1 \Phi F' \Phi(\eta) \varphi_n d\eta}{\int_0^1 F' \Phi(\eta) \varphi_n^2 d\eta} = \frac{2}{\varepsilon_n} \ \frac{Q/2^\alpha - \Phi(1) \int_0^1 \varphi_n f_1 \Phi(\eta) d\eta / \varphi_n(1)}{\left(\frac{\partial \varphi'}{\partial \varepsilon}\right)_{\substack{\eta=1 \\ \varepsilon=\varepsilon_n}} - \frac{\mathrm{Pe}_v}{2}\left(\frac{\partial \varphi}{\partial \varepsilon}\right)_{\substack{\eta=1 \\ \varepsilon=\varepsilon_n}}},$$

will be a solution of (7).

The solution of (15) has the form

$$\psi(\eta) = \frac{2}{\mathrm{Pe}_v \Phi(1)} \int_0^1 f_1 \Phi(\eta) d\eta - \frac{2^{1-\alpha} Q}{\mathrm{Pe}_v} - \int_\eta^1 \frac{d\lambda}{\Phi(\lambda) \lambda^\alpha \gamma} \int_0^\lambda f \Phi(\xi) d\xi. \tag{17}$$

Using (17), we determine the Nusselt number in the domain of developed heat transfer

$$\mathrm{Nu}_* = \frac{2\psi'(1)}{\psi(1)} = \mathrm{Pe}_v + 2Q\Phi(1) \left\{ \int_0^1 \left[(Q+1) F^2 - F \int_0^\eta (2\lambda)^\alpha f d\lambda \right] \frac{\Phi(\eta) d\eta}{\eta^\alpha \gamma} \right\}^{-1}. \tag{18}$$

For $\mathrm{Pe}_v = 0$, both (17) and (18) go over into the appropriate expressions for the temperature field and Nu_* in tubes with impermeable walls for a given heat flux and the presence of internal heat sources [8]. In the case $Q \to \infty$, solutions for heat transfer with suction and injection follow from (17) and (18) in the absence of heat liberation in the stream [3, 5]. For $Q = -1$, when the average mass temperature and the wall temperature do not vary along the tube length, (17) and (18) agree with (10) and (11).

Asymptotic formulas for the temperature field and Nusselt number can be obtained for strong injection and suction ($|\mathrm{Pe}_v| \gg 1$). For injection the first terms describing heat transmission by heat conduction can be omitted in (15) with the exception of the domain near the axis. Without taking this term and the correspodning boundary condition $\psi''(0) = 0$ into account, we obtain the approximate temperature distribution for strong injection which is valid over the whole tube section except for a narrow zone near the axis:

$$\psi(\eta) = \frac{2}{|\mathrm{Pe}_v|}\left(\int_\eta^1 \frac{\lambda^\alpha f}{F} d\lambda + \frac{Q+1}{2^\alpha} \ln F + \frac{Q}{2^\alpha} \right) f(1).$$

Taking account of $F(1) = \gamma(1) = 1$, we obtain the following asymptotic expression

$$\frac{\psi(1) - \psi(\eta)}{\psi(1)} = 1 - \exp\left[\frac{\mathrm{Pe}_v}{2}(\eta - 1)\right]$$

for suction from (17), from which it is seen that for strong suction the temperature is practically constant in the whole tube section while a narrow thermal boundary layer (of thickness $O(1/\mathrm{Pe}_v)$) in which a sharp temperature variation occurs, is formed at the wall. We have correspondingly from (18)

$$\mathrm{Nu}_* = -\frac{2^{1+\alpha} f(1)}{Q} + \left[\frac{2^\alpha f'(1) - F''(1)}{Q} - F''(1)\right] \frac{4}{\mathrm{Pe}_v} \quad \text{for} \quad \mathrm{Pe}_v \to -\infty,$$

$$\mathrm{Nu}_* = \mathrm{Pe}_v \left\{ 1 + \sqrt{\frac{\mathrm{Pe}_v \mu}{\pi \gamma(0)}} \left(\frac{\mathrm{Pe}_v \mu \pi}{16 \gamma(0)} \right)^{\alpha/2} \frac{Q\Phi(1)}{[(Q+1)\mu - f(0)]} \right\} \text{for} \quad \mathrm{Pe}_v \to \infty,$$

where

$$\mu = \lim_{\eta \to 0} F(\eta)/\eta^{1+\alpha}.$$

The Nusselt number determined according to (18) depends on Q exactly as for flows in impermeable tubes [8], and consequently, can take on negative values, which is a substantial disadvantage. Hence, it is expedient to determine Nu, analogously to [8], in such a way that there would be no dependence of Nu on Q, which will significantly facilitate performance of practical computations. To this end, let us represent (17) in the form

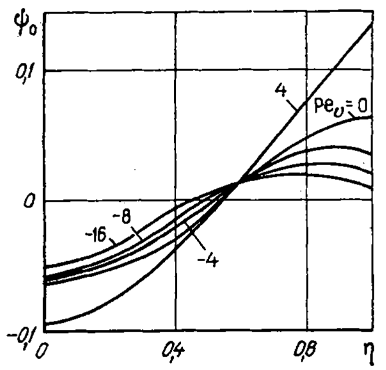

Fig. 3

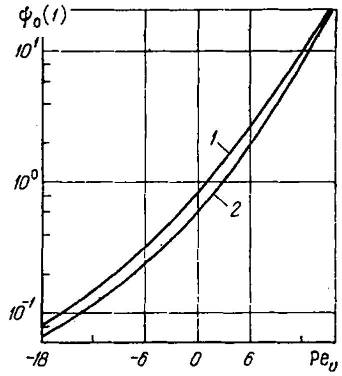

Fig. 4

Fig. 3. Temperature distribution over the section of a circular tube under adiabatic conditions.

Fig. 4. Dependence of the adiabatic wall temperature on the parameter Pe_v: 1) $\alpha = 0$; 2) 1.

$$\psi(\eta) = \frac{2}{Pe_v \Phi(1)} \int_0^1 \frac{F' - (2\eta)^\alpha f}{2^\alpha} \Phi(\eta)\, d\eta - \int_\eta^1 \frac{d\lambda}{\Phi(\lambda)\lambda^\alpha \gamma} \int_0^\lambda \frac{F' - (2\xi)^\alpha}{2^\alpha} \Phi(\xi)\, d\xi +$$

$$+ Q\left[\frac{2}{Pe_v \Phi(1)} \int_0^1 \frac{F'}{2^\alpha} \Phi(\eta)\, d\eta - \int_\eta^1 \frac{d\lambda}{\Phi(\lambda)\lambda^\alpha \gamma} \int_0^\lambda \frac{F'}{2^\alpha} \Phi(\xi)\, d\xi\right] = \psi_0(\eta) + Q\psi_1(\eta). \quad (19)$$

Here $\psi_0(\eta)$ is the temperature field for $Q = 0$, when the tube wall is adiabatic, and $\psi_1(\eta)$ is the temperature in the absence of internal heat sources. The temperature distribution $\psi_0(\eta)$ over the section of a circular tube, and the dependence of the adiabatic wall temperature $\psi_0(1)$ on the parameter Pe_V computed under conditions (12) are shown, respectively, in Figs. 3 and 4. Injection substantially reduces, while suction increases the stream and wall temperatures, where as follows from Fig. 3, the maximum temperature during injection is shifted from the wall to the stream.

Taking (19) into account, we determine the Nusselt number in terms of the function $\psi_1(\eta)$

$$Nu_{*1} = 2\psi_1'(1)/\psi_1(1) = Pe_v + \frac{2}{\psi(1) - \psi_0(1)}. \quad (20)$$

In this case Nu_{*1} is independent of Q and agrees with the formulas obtained in [3, 5] for $q_V = 0$, and the lines 3 and 4 shown in Fig. 2, respectively, for the plane and circular pipe with a Poiseuille velocity profile. To compute the wall temperature $\theta_{*w} = \theta_m + \psi(1)$ for given values of Q and Pe_V, it is sufficient to know the dependence of Nu_{*1} and $\psi_0(1)$ on Pe_V as follows from (14) and (20).

NOTATION

x, r, longitudinal and radial coordinates; ρ, density; c_p, specific heat; λ_T, coefficients of molecular and turbulent heat conduction; ν, kinematic viscosity; r_0, tube radius; U_0, mean velocity at tube entrance; V, suction or injection rate; q_V, density of internal heat sources; $Pe_V = 2r_0V/\nu$; $\gamma = 1 + \lambda_T/\lambda$; $\alpha = 0$, for a plane, and $\alpha = 1$ for a circular tube.

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METHODOLOGICAL PECULIARITIES OF SHORT MEASUREMENTS AT THE STAGE OF IRREGULAR THERMAL REGIME

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The article discusses the peculiarities of thermophysical measurements when low-inertia heaters are used; it also presents the results of measurements of thermal activity.

The present technical state and the technology of applying thin conducting films open up broad possibilities of using them in thermophysical experiments. Particularly promising is their use as low-inertia elements for short impulse measurements at the stage of irregular thermal regime. With their aid it is possible to study the thermophysical characteristics of such objects as liquids, gases, or solids that ensure adhesion to the layers that had been vaporized on. Calculations show [1] that when vaporized-on resistor elements (RE) are used, the influence of the proper heat capacity of the element on the results of measuring the thermal activity of a solid or liquid is practically insignificant already when impulse measurements last 10^{-5}-10^{-4} sec. When elements with extremely small thickness (70–80 Å) are used, this time can be reduced to 10^{-6} sec.

In short-term thermophysical experiments, thin metal filaments can be used together with vaporized-on layers. The technology of making such filaments has by now been well mastered. Evaluations of the influence of the proper heat capacity of a filament on the results of the measurement of thermal conductivity show [2] that for a filament with radius 10^{-6} m the influence of the heat capacity is small already when the impulses last so much as 10^{-3} sec.

Since the measurements are short, the method is bound to have a number of advantages. In particular, favorable conditions are created for using thermal measurements to diagnose dynamic processes. Some experience in using thermal diagnostics for studying chemical reactions, phase transformations, diffusion and other processes has already found expression in various works [3-8].

The smallness of the spatial region in which the temperature field is non-steady-state is another favorable feature of these measurements: heat transfer occurring under such specific conditions reflects the molecular heat transfer that is only slightly distorted by radiation [9, 10]. In consequence, short measurements have an advantage as a matter of principle as compared with a number of other methods, in particular, steady-state methods.

On the other hand, if methods of short measurement are to be applied correctly, we must examine the peculiarities of this application occasioned by the small length of diffusion of the temperature field into the investigated medium, by the considerable temperature gradient, etc.

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